Time variation of the fine structure constant in decrumpling or TVSD model

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(February 1, 2008)

Abstract

Within the framework of a model universe with time variable space dimension (TVSD), known as decrumpling or TVSD model, we study the time variation of the fine structure constant. Using observational bounds on the present time variation of the fine structure constant, we are able to obtain an upper limit for the absolute value of the present time variation of spatial dimensions.

Although time variability of spatial dimensions have not been firmly achieved in experiments and theories, such dynamical behavior of the spatial dimensions should not be ruled out in the context of cosmology and astroparticle physics [1]-[7].

In this letter, we study the time variation of the fine structure constant in decrumpling or TVSD model.[2]

Assume the universe consists of a fixed number $\bar{N}$ of universal cells having a characteristic length δ in each of their dimensions. The volume of the universe at the time t depends on the configuration of the cells. It is easily seen that [7]

$$\text{vol}_{D_t}(\text{cell}) = \text{vol}_{D_0}(\text{cell})\delta^{D_t - D_0}, \tag{1}$$

where the t subscript in D_t means D is as a function of time. Interpreting the radius of the universe, a, as the radius of gyration of a crumpled "universal surface", the volume of space can be written [7]

$$
\begin{aligned}
a^{D_t} &= \bar{N}\text{vol}_{D_t}(\text{cell}) \\
&= \bar{N}\text{vol}_{D_0}(\text{cell})\delta^{D_t - D_0} \\
&= a_0^{D_0}\delta^{D_t - D_0}
\end{aligned}
\tag{2}
$$

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[2]It is worth mentioning that from Eq.(1) to Eq.(19) we use a natural unit system that sets k_B, c and $\hbar$ all equal to 1, so that $\ell_P = M_P^{-1} = \sqrt{G}$. From Eq.(20) to Eq.(54) we use the International System of Units.

or
$$\left(\frac{a}{\delta}\right)^{D_t} = \left(\frac{a_0}{\delta}\right)^{D_0} = e^C, \tag{3}$$

where C is a universal positive constant. Its value has a strong influence on the dynamics of spacetime, for example on the dimension of space, say, at the Planck time. Hence, it has physical and cosmological consequences and may be determined by observations. The zero subscript in any quantity, e.g. in a_0 and D_0, denotes its present values. We coin the above relation as a "dimensional constraint" which relates the "scale factor" of the model universe to the space dimension. In our formulation, we consider the comoving length of the Hubble radius at present time to be equal to one. So the interpretation of the scale factor as a physical length is valid. The dimensional constraint can be written in this form

$$\frac{1}{D_t} = \frac{1}{C} \ln\left(\frac{a}{a_0}\right) + \frac{1}{D_0}. \tag{4}$$

It is seen that by expansion of the universe, the space dimension decreases. Time derivative of Eqs.(3) or (4) leads to

$$\dot{D}_t = -\frac{D_t^2 \dot{a}}{Ca}. \tag{5}$$

It can be easily shown that the case of constant space dimension corresponds to when C tends to infinity. In other words, C depends on the number of fundamental cells. For $C \to +\infty$, the number of cells tends to infinity and $\delta \to 0$. In this limit, the dependence between the space dimensions and the radius of the universe is removed, and consequently we have a constant space dimension.

We define D_P as the space dimension of the universe when the scale factor is equal to the Planck length ℓ_P. Taking $D_0 = 3$ and the scale of the universe today to be the present value of the Hubble radius H_0^{-1} and the space dimension at the Planck length to be $4, 10$, or 25, from Kaluza-Klein and superstring theories, we can obtain from Eqs. (3) and (4) the corresponding value of C and δ

$$\frac{1}{D_P} = \frac{1}{C} \ln\left(\frac{\ell_P}{a_0}\right) + \frac{1}{D_0} = \frac{1}{C} \ln\left(\frac{\ell_P}{H_0^{-1}}\right) + \frac{1}{3}, \tag{6}$$

$$\delta = a_0 e^{-C/D_0} = H_0^{-1} e^{-C/3}. \tag{7}$$

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Table 1: Values of C and δ for some values of D_P [1]-[7]. Time variation of space dimension today has also been calculated in terms of yr^{-1}.

D_P	C	δ (cm)	$\dot{D}_t\|_0$ (yr^{-1})
3	$+\infty$	0	0
4	1678.797	8.6158×10^{-216}	$-5.4827 \times 10^{-13} h_0$
10	599.571	1.4771×10^{-59}	$-1.5352 \times 10^{-12} h_0$
25	476.931	8.3810×10^{-42}	$-1.9299 \times 10^{-12} h_0$
$+\infty$	419.699	ℓ_P	$-2.1931 \times 10^{-12} h_0$

In Table 1, values of C, δ and also $\dot{D}_t\|_0$ for some interesting values of D_P are given. These values are calculated by assuming $D_0 = 3$ and $H_0^{-1} = 3000 h_0^{-1} \mathrm{Mpc} = 9.2503 \times 10^{27} h_0^{-1} \mathrm{cm}$, where $h_0 = 0.68 \pm 0.15$. Since the value of C and δ are not very sensitive to h_0 we take $h_0 = 1$.

Let us define the action of the model for the special Friedmann-Robertson-Walker (FRW) metric in an arbitrary fixed space dimension D, and then try to generalize it to variable dimension D_t. Now, take the metric in constant $D + 1$ spacetime dimensions in the following form

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\Sigma_k^2, \tag{8}$$

where $N(t)$ denotes the lapse function and $d\Sigma_k^2$ is the line element for a D-manifold of constant curvature $k = +1, 0, -1$. The Ricci scalar is given by

$$R = \frac{D}{N^2}\left\{ \frac{2\ddot{a}}{a} + (D-1)\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{N^2 k}{a^2} \right] - \frac{2\dot{a}\dot{N}}{aN} \right\}. \tag{9}$$

Substituting from Eq.(9) in the Einstein-Hilbert action for pure gravity,

$$S_G = \frac{1}{2\kappa} \int d^{(1+D)}x\sqrt{-g}R, \tag{10}$$

and using the Hawking-Ellis action of a perfect fluid for the model universe with variable space dimension the following Lagrangian has been obtained for decrumpling or TVSD model (see Ref.[7])

$$L_I := -\frac{V_{D_t}}{2\kappa N}\left(\frac{a}{a_0}\right)^{D_t} D_t(D_t - 1)\left[\left(\frac{\dot{a}}{a}\right)^2 - \frac{N^2 k}{a^2} \right] - \rho N V_{D_t}\left(\frac{a}{a_0}\right)^{D_t}, \tag{11}$$

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where $\kappa = 8\pi M_P^{-2} = 8\pi G$, ρ the energy density, and V_{D_t} the volume of the space-like sections

$$V_{D_t} = \frac{2\pi^{(D_t+1)/2}}{\Gamma[(D_t+1)/2]}, \quad \text{closed Universe, } k = +1, \tag{12}$$

$$V_{D_t} = \frac{\pi^{(D_t/2)}}{\Gamma(D_t/2+1)}\chi_c^{D_t}, \quad \text{flat Universe, } k = 0, \tag{13}$$

$$V_{D_t} = \frac{2\pi^{(D_t/2)}}{\Gamma(D_t/2)}f(\chi_c), \quad \text{open Universe, } k = -1. \tag{14}$$

Here χ_C is a cut-off and $f(\chi_c)$ is a function thereof (see Ref. [7]).

In the limit of constant space dimensions, or $D_t = D_0$, L_I approaches to the Einstein-Hilbert Lagrangian which is

$$L_I^0 := -\frac{V_{D_0}}{2\kappa_0 N}\left(\frac{a}{a_0}\right)^{D_0} D_0(D_0 - 1)\left[\left(\frac{\dot{a}}{a}\right)^2 - \frac{N^2 k}{a^2}\right] - \rho N V_{D_0}\left(\frac{a}{a_0}\right)^{D_0}, \tag{15}$$

where $\kappa_0 = 8\pi G_0$ and the zero subscript in G_0 denotes its present value. So, Lagrangian L_I cannot abandon Einstein's gravity. Varying the Lagrangian L_I with respect to N and a, we find the following equations of motion in the gauge $N = 1$, respectively

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{2\kappa\rho}{D_t(D_t - 1)}, \tag{16}$$

$$(D_t - 1)\left\{\frac{\ddot{a}}{a} + \left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right]\left(-\frac{D_t^2}{2C}\frac{d\ln V_{D_t}}{dD_t} - 1 - \frac{D_t(2D_t - 1)}{2C(D_t - 1)} + \frac{D_t^2}{2D_0}\right)\right\}$$

$$+\kappa p\left(-\frac{d\ln V_{D_t}}{dD_t}\frac{D_t}{C} - \frac{D_t}{C}\ln\frac{a}{a_0} + 1\right) = 0. \tag{17}$$

Using (5) and (16), the evolution equation of the space dimension can be obtained by

$$\dot{D}_t^2 = \frac{D_t^4}{C^2}\left[\frac{2\kappa\rho}{D_t(D_t - 1)} - k\delta^{-2}e^{-2C/D_t}\right]. \tag{18}$$

The continuity equation of decrumpling or TVSD model can be obtained by (16) and (17)

$$\frac{d}{dt}\left[\rho\left(\frac{a}{a_0}\right)^{D_t}V_{D_t}\right] + p\frac{d}{dt}\left[\left(\frac{a}{a_0}\right)^{D_t}V_{D_t}\right] = 0. \tag{19}$$

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Let us now study the time variation of the fine structure constant in decrumpling or TVSD model. In the International System of Units, the fine structure constant is given by $\alpha \equiv \frac{e^2}{4\pi\epsilon_0 \hbar c} \simeq \frac{1}{137}$ and in the Heaviside-Lorentz System $\alpha \equiv \frac{e^2}{\hbar c} \simeq \frac{1}{137}$. It is argued in [8] that in D-dimensional spaces the dimensionless constant of Nature in the Heaviside-Lorentz System is proportional to $h^{2-D}e^{D-1}G^{\frac{3-D}{2}}c^{D-4}$. From this for $D = 3$ we obtain $\frac{e^2}{\hbar c}$. The dimensionless quantity $e^2/(hc)$ was first emphasized by Sommerfeld [9].

The fine structure constant in D-dimensional space is given by (See Appendix)

$$\hat{\alpha} = e^{(D-1)}\hat{k}^{\frac{(D-1)}{2}}\hat{G}^{(\frac{3-D}{2})}\hbar^{(2-D)}c^{(D-4)}. \tag{20}$$

In decrumpling or TVSD model, the definition of the fine structure constant in terms of Planck charge is (see Eq.(29) in Appendix)

$$\hat{\alpha} \equiv \left(\frac{e}{\hat{Q}_P}\right)^{(D_t-1)}. \tag{21}$$

By using the same approach given in Appendix, one can easily obtain the fine structure constant in decrumpling or TVSDmodel (see Eq.(54) in Appendix and Eq.(20))

$$\hat{\alpha} = e^{(D_t-1)}\hat{k}^{\frac{(D_t-1)}{2}}\hat{G}^{(\frac{3-D_t}{2})}\hbar^{(2-D_t)}c^{(D_t-4)}. \tag{22}$$

Time derivative of this equation leads to[3]

$$\frac{\dot{\hat{\alpha}}}{\hat{\alpha}} = \frac{(3-D_t)}{2}\frac{\dot{\hat{G}}}{\hat{G}} + \dot{D}_t \ln\left(\frac{e}{\hat{Q}_P}\right). \tag{23}$$

Using this equation at the present time and taking $D_t|_0 = 3$ we are led to

$$
\begin{aligned}
\left.\frac{\dot{\hat{\alpha}}}{\hat{\alpha}}\right|_0 &= \left.\dot{D}_t\right|_0 \ln\left(\frac{e}{\sqrt{4\pi\epsilon_0 \hbar c}}\right) \\
&\simeq \frac{1}{2}\left.\dot{D}_t\right|_0 \ln\frac{1}{137}
\end{aligned}
\tag{24}
$$

From this equation, it can be easily seen that $\dot{\hat{\alpha}}/\hat{\alpha}$ has positive value ($\dot{D}_t < 0$). This tells us that the value of the fine structure constant increases within the

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[3]We take the quantities e, $\hat{\kappa}$, $\hbar$, and c to be constant.

cosmic time in TVSD or decrumpling model. Atomic clocks are one of the principal methods we have to measure possible variation of the fine structure constant on Earth. The latest constraint on possible time variation of alpha, using atomic clocks, is [12] (see also Refs.[13, 14])

$$\left|\frac{\dot{\alpha}}{\alpha}\right|_0 < 10^{-15}\text{yr}^{-1}. \tag{25}$$

Using Eqs.(24) and (25) we are led to

$$\left|\dot{D}_t\right|_0 < 10^{-15}\text{yr}^{-1}. \tag{26}$$

This result leads to an upper limit for the absolute value of the present time variation of the spatial dimension in decrumpling or TVSD model.

Appendix

Let us now obtain a general formula for the fine structure constant in D-dimensional spaces in the International System of Units. To obtain a general formula for the fine structure constant in D-dimensional spaces we use the definition of the fine structure constant in terms of Planck charge Q_P

$$Q_P \equiv \sqrt{4\pi\epsilon_0\hbar c}. \tag{27}$$

In 3-dimensional spaces, the definition of the fine structure constant in terms of Planck charge is

$$\alpha \equiv \left(\frac{e}{Q_P}\right)^2. \tag{28}$$

In D-dimensional spaces, a generalized form of (28) defines the fine stucture constant. Defining $\hat{\alpha}$ as the fine structure constant in D-dimensional spaces, we have

$$\hat{\alpha} \equiv \left(\frac{e}{\hat{Q}_P}\right)^{(D-1)}, \tag{29}$$

where $\hat{Q}_P$ is Planck charge in D-dimensional spaces and as a function of D. To obtain the fine stucture constant in higher dimensions, we must first

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obtain Planck charge in higher dimensions. In doing so, we propose that in D-dimensional spaces, Planck charge can be written in terms of four fundamental constants

$$\hat{k}, \quad \hat{G}, \quad \hbar, \quad c,$$

where $\hat{k}$ is the electrostatic coupling constant and $\hat{G}$ is the gravitational constant in D-dimensional spaces. We also define four unknown functions of the space dimension

$$\beta(D), \quad \eta(D), \quad \xi(D), \quad \tau(D),$$

so that these four unknown functions of D are the exponents of $\hat{k}$, $\hat{G}$, $\hbar$ and c in the formula of Planck charge

$$\hat{Q}_P = \hat{k}^{\beta(D)} \hat{G}^{\eta(D)} \hbar^{\xi(D)} c^{\tau(D)}. \tag{30}$$

Coulomb's law for the electrostatic force in D-space and 3-space are defined by

$$F_D = \frac{\hat{k} q_1 q_2}{r^{D-1}}, \tag{31}$$

$$F = \frac{k q_1 q_2}{r^2}, \tag{32}$$

where $k = 1/(4\pi\epsilon_0)$. Newton's law for the gravitational force in D-space and 3-space are defined by

$$F_D = \frac{\hat{G} m_1 m_2}{r^{D-1}}, \tag{33}$$

$$F = \frac{G m_1 m_2}{r^2}. \tag{34}$$

Using Gauss law in D-dimensional spaces, one can derive (see Ref. [10] for more detailed explanation)

$$G = \frac{S_D}{4\pi} \frac{\hat{G}}{V_{(D-3)}}, \tag{35}$$

and

$$k = \frac{S_D}{4\pi} \frac{\hat{k}}{V_{(D-3)}}, \tag{36}$$

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where

$$S_D \equiv \frac{2\pi^{D/2}}{\Gamma\left(\frac{D}{2}\right)}, \tag{37}$$

is the surface area of the unit sphere in D-dimensional spaces and $V_{(D-3)}$ is the volume of $(D-3)$ extra spatial dimensions. For $D = 3$, we have $S_3 = 4\pi$. The units of $\hat{k}$ and $\hat{G}$ can be easily found by (36) and (35), respectively. Moreover, in D-space the units of $\hbar$ and c is the same as their units in 3-space. So, in terms of mass (M), length (L), time (T) and electric charge (Q), in D-dimensional spaces, we have the following units for $\hat{k}$, $\hat{G}$, $\hbar$ and c

$$\left[\hat{k}\right] = Q^{-2}ML^DT^{-2}, \tag{38}$$
$$\left[\hat{G}\right] = M^{-1}L^DT^{-2}, \tag{39}$$
$$[\hbar] = ML^2T^{-1}, \tag{40}$$
$$[c] = LT^{-1}. \tag{41}$$

We also know that the units of Planck charge $\hat{Q}_P$ is equal to the unit of electric charge, Q,

$$\left[\hat{Q}_P\right] = Q. \tag{42}$$

Therefore, we can rewrite (30) in terms of units

$$Q = \left(Q^{-2}ML^DT^{-2}\right)^{\beta(D)} \left(M^{-1}L^DT^{-2}\right)^{\eta(D)} \left(ML^2T^{-1}\right)^{\xi(D)} \left(LT^{-1}\right)^{\tau(D)}. \tag{43}$$

To satisfy (43) for the units of Planck charge in D-dimensional spaces, in the right-hand side of (43) the exponents of M must be vanished

$$\beta - \eta + \xi = 0, \tag{44}$$

the exponents of L must be vanished

$$D\beta + D\eta + 2\xi + \tau = 0, \tag{45}$$

the exponents of T must be vanished

$$2\beta + 2\eta + \xi + \tau = 0, \tag{46}$$

and finally the exponents of Q must be equal to one

$$-2\beta = 1. \tag{47}$$

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Using four equations (44)-(47) we can obtain four functions β, η, ξ and τ with respect to D

$$\beta(D) = -\frac{1}{2}, \tag{48}$$

$$\eta(D) = \frac{D-3}{2(D-1)}, \tag{49}$$

$$\xi(D) = \frac{D-2}{D-1}, \tag{50}$$

$$\tau(D) = \frac{4-D}{D-1}. \tag{51}$$

Substituting (48)-(51) in (30) we obtain Planck charge as a function of D in D-dimensional spaces

$$\hat{Q}_P = \hat{k}^{-\frac{1}{2}}\hat{G}^{\frac{D-3}{2(D-1)}}\hbar^{\frac{D-2}{D-1}}c^{\frac{4-D}{D-1}}. \tag{52}$$

Now, substituting (52) in (29) yields the fine structure constant in D-dimensional spaces

$$\hat{\alpha} = \left(\frac{e}{\hat{Q}_P}\right)^{(D-1)} = \left(\frac{e}{\hat{k}^{-\frac{1}{2}}\hat{G}^{\frac{D-3}{2(D-1)}}\hbar^{\frac{D-2}{D-1}}c^{\frac{4-D}{D-1}}}\right)^{(D-1)}, \tag{53}$$

or

$$\hat{\alpha} = e^{(D-1)}\hat{k}^{\frac{(D-1)}{2}}\hat{G}^{(\frac{3-D}{2})}\hbar^{(2-D)}c^{(D-4)}. \tag{54}$$

Eq.(54) gives us the fine structure constant in D-dimensional spaces and in the International System of Units and reads $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$ in 3-dimensional spaces. It is worth mentioning that in Ref.[11] by using another approach the fine structure constant in D-dimensional space has been obtained. Our result here, i.e. Eq.(54), is in agreement with the result presented in Ref.[11].

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